International Journal of Mathematics and Physical Sciences Research ISSN 2348-5736 (Online)
Vol. 4, Issue 1, pp: (67-73), Month: April 2016 - September 2016, Available at: www.researchpublish.com

Generalized Contraction-Type Mappings on Multiplicative – Metric Space

[1]Kamal Kumar, [2]Nisha Sharma, [3]Mamta Rani, [4]Rajeev Jha

[1]Department of Mathematics, Pt. JLN Govt. College Faridabad, Sunrise University Alwar
[2]Department of Mathematics, Manav Rachna International University Faridabad
[3]Department of Mathematics, Pt. JLN Govt. College Faridabad
[4]Department of Mathematics, Teerthankar Mahaveer University Moradabad(UP)

Abstract: **In this paper, we generalized some unique fixed point theorems in the context of multiplicative metric spaces. We note that some fixed point theorems can be deduced in multiplicative metric space by using the established results. we used the concept of multiplicative contraction mapping and proved some fixed point theorems of such mappings on complete multiplicative metric spaces.**
MSC: **54E40, 54E35, 54H25.**

Keywords: **Complex-valued metric space, Multi-valued mappings, Contraction mappings, Common fixed point.**

1. INTRODUCTION

1.1. The notion of complex-valued metric spaces was introduced by Azam et al. [2]. Ozavsar and Cevikel [4] initiated the concept of the multiplicative metric space in such a way that the usual triangular inequality is replaced by "Multiplicative triangle inequality $d(x, y) \leq d(x, z).d(z, y)$ for all $x, y, z \in X$". It is well known fact that the mathematical results regarding fixed points of contraction-type mappings are very useful for determining the existence and uniqueness of solutions to various mathematical models. The study of fixed points of mappings satisfying certain contraction conditions has many applications and has been at the centre of various research activities. we introduce concept of multiplicative contraction mapping and prove some fixed point theorems of multiplicative contraction mappings on multipliactive metric spaces. The study of fixed points of mappings satisfying certain contraction conditions has many applications and has been at the centre of various research activities.The theory of fixed points has been developed, regarding the results to finding the fixed points over the last 45 years. . It is well known that the set of positive real numbers R_+ is not complete according to the usual metric. To emphasize the importance of this study, we should first note that R+ is a complete multiplicative metric space with respect to the multiplicative metric.

Some important definitions are as follows:

Definition 1.2. Let X be a nonempty set. Multiplicative metric [4] is a mapping $d : X \times X \to \mathbb{R}_+$ satisfying the following conditions:

(m1) $d(x,y) \geq 1$ for all $x, y \in X$ and $d(x, y) = 1$ if and only if $x = y$,

(m2) $d(x, y) = d(y, x)$ for all $x, y \in X$,

(m3) $d(x, z) \leq d(x, y)\cdot d(y, z)$ for all $x, y, z \in X$ (multiplicative triangle inequality).

2. MULTIPLICATIVE METRIC TOPOLOGY

We shall begin this section with the following examples, which will be used throughout the paper.

Definition 1.3.[4]: Let (X, d) be a multiplicative metric space, $x \in X$ and $\varepsilon > 1$. We now define a set $B_\varepsilon(x) = \{y \in X \mid d(x, y) < \varepsilon$,which is called multiplicative open ball of radius ε with center x. Similarly, one can describe multiplicative closed ball as $B_\varepsilon(x) = \{y \in X \mid d(x, y) \leq \varepsilon\}$.

Definition 1.4.[4] : Let (X, d) be a multiplicative metric space and $A \subset X$. Then we call $x \in A$ a multiplicative interior point of A if there exists an $\varepsilon > 1$ such that $B\varepsilon(x) \subset A$. The collection of all interior points of A is called multiplicative interior of A and denoted by int (A).

Definition 1.5. [4] : Let (X, d) be a multiplicative metric space and $A \subset X$. If every point of A is a multiplicative interior point of A, i.e., A = int (A), then A is called a multiplicative open set.

Lemma 1.6. Let (X, d) be a multiplicative metric space. Each multiplicative open ball of X is a multiplicative open set. **Proof.** Let $x \in X$ and $B_\varepsilon(x)$ be a multiplicative open ball. For $y \in B_\varepsilon(x)$, if we let $\delta = \frac{\varepsilon}{d(x,y)}$ and $z \in B_\delta(y)$,

then $d(y, z) < \frac{\varepsilon}{d(x,y)}$, from which we conclude that $d(x, z) < d(x, y) \cdot d(y, z) < \varepsilon$.

This shows that $z \in B_\varepsilon(x)$, which means that $B_\delta(y) \subset B_\varepsilon(x)$ i.e. y is interior point of $B_\varepsilon(x)$. Thus $B_\varepsilon(x)$ is multiplicative open set.

Lemma 1.7. The intersection of any finite family of multiplicative open sets is also a multiplicative open set. **Proof.** Let B_1 and B_2 be two multiplicative open sets and $y \in B_1 \cap B_2$. Then there are $\delta_1, \delta_2 > 1$ such that $B_{\delta 1}(y) \subset B_1$ and $B_{\delta 2}(y) \subset B_2$. Letting δ be the smaller of δ_1 and δ_2, we conclude that $B_\delta(y) \subset B_1 \cap B_2$. Hence the intersection of any finite family of multiplicative open sets is a multiplicative open set.

Definition 1.8. [4] Let (X, d) be a multiplicative metric space. A point $x \in X$ is said to be multiplicative limit point of $S \subset X$ if and only if $(B_\varepsilon(x) \setminus \{x\}) \cap S \neq \phi$ for every $\varepsilon > 1$. The set of all multiplicative limit points of the set S is denoted by S'.

Definition 1.9. [4] Let (X, d) be a multiplicative metric space. We call a set $S \subset X$ multiplicative closed in (X, d) if S contains all of its multiplicative limit points.

Definition 1.10.[4] : Let (X, d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x \in X$. If for every multiplicative open ball $B_\varepsilon(x)$, there exists a natural number $\mathbb{N}$ such that

$n \geq \mathbb{N} \Rightarrow x_n \in B_\varepsilon(x)$, then the sequence (x_n) is said to be multiplicative convergent to x, denoted by $x_n \to x$ $(n \to \infty)$.

Lemma 1.11. [4] Let (X, d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x \in X$. Then $x_n \to x$ $(n \to \infty)$ if and only if $d(x_n, x) \to 1$ $(n \to \infty)$.

Proof. Suppose that the sequence $\{x_n\}$ is multiplicative convergent to x. That is, for every $\varepsilon > 1$, there is a natural number N such that $d(x_n, x) < \varepsilon$ whenever $n \geq N$. Thus we have the following inequality

$1/\varepsilon < d(x_n, x) < 1 \cdot \varepsilon$ for all $n \geq N$.

This means $|d(x_n, x)| < \varepsilon$ for all $n \geq N$, which implies that the sequence $d(x_n, x)$ is multiplicative convergent to 1. It is clear to verify the converse.

Lemma 1.12. [9] Let (X, d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X. If the sequence $\{x_n\}$ is multiplicative convergent, then the multiplicative limit point is unique.

Proof. Let $x, y \in X$ such that $x_n \to_* x$ and $x_n \to_* y$ $(n \to \infty)$. That is, for every $\varepsilon > 1$, there exists N such that, for all $n \geq N$, we have $d(x_n, x) < \sqrt{\varepsilon}$ and $d(x_n, y) < \sqrt{\varepsilon}$. Then we have $d(x, y) \leq d(x_n, x) \cdot d(x_n, y) < \varepsilon$

Since ε is arbitrary, $d(x, y) = 1$. This says $x = y$.

Theorem 1.13. [9] Let (X, d_X) and (Y, d_Y) be two multiplicative metric spaces, $f : X \to Y$ be a mapping and $\{x_n\}$ be any sequence in X. Then f is multiplicative continuous at the point $x \in X$ if and only if $f(x_n) \to f(x)$ for every sequence $\{x_n\}$ with $x_n \to x$ $(n \to \infty)$.

Proof. Suppose that f is multiplicative continuous at the point x and $x_n \to x$. From the multiplicative continuity of f, we have that, for every $\varepsilon > 1$, there exists $\delta > 1$ such that

$f(B_\delta(x)) \subset B_\varepsilon(f(x))$. Since $x_n \to x$ $(n \to \infty)$, there exists N such that $n \geq N$ implies $x_n \in B_\delta(x)$. By virtue of the above inclusion, then $f(x_n) \in B_\varepsilon(f(x))$ and hence $f(x_n) \to f(x)$ $(n \to \infty)$.

Conversely, assume that f is not multiplicative continuous at x. That is, there exists an $\varepsilon > 1$ such that, for each $\delta > 1$, we have $x' \in X$ with $d_X(x', x) < \delta$ but

$d_Y(f(x'), f(x)) \geq \varepsilon$…………..(1.1)

Now, take any sequence of real numbers (δ_n) such that $\delta_n \to 1$ and $\delta_n > 1$ for each n. For each n, select x′ that satisfies the equation (1.1) and call this x_n'. It is clear that $x_n' \to x$, but $f(x_n')$ is not multiplicative convergent to f(x). Hence we see that if f is not multiplicative continuous, then not every sequence $\{x_n\}$ with $x_n \to x$ will yield a sequence $f(x_n) \to f(x)$. Taking the contrapositive of this statement demonstrates that the condition is sufficient.

Similarly, we can prove the following theorem.

Theorem 1.14. Let (X, d) be a multiplicative metric space and $\{x_n\}$ be a sequence in X. The sequence is multiplicative convergent, then it is a multiplicative Cauchy sequence.

Proof. Let x ∈ X such that $x_n \to x$. Hence we have that for any $\varepsilon > 1$, there exist a natural number N such that $d(x_n, x) < \sqrt{\varepsilon}$ and $d(x_m, x) < \sqrt{\varepsilon}$ for all $m, n \geq N$.

By the multiplicative triangle inequality, we get

$d(x_n, x_m) \leq d(x_n, x) \cdot d(x, x_m) < \sqrt{\varepsilon} \cdot \sqrt{\varepsilon} = \varepsilon$,

which implies $\{x_n\}$ is a multiplicative Cauchy sequence.

Theorem 1.15. (Multiplicative characterization of supremum) Let A be a nonempty subset of $\mathbb{R}_+$. Then s = sup A if and only if

(i) $a \leq s$ for all a ∈ A

(ii) there exists at least a point a ∈ A such that $|s / a| < \varepsilon$ for all $\varepsilon > 1$.

Proof. Let s = sup A. Then from the definition of supremum, the condition (1) is trivial. To prove the condition (1), assume that there is an $\varepsilon > 1$ such that there are no elements a ∈ A such that $|s / a| < \varepsilon$. If this is the case, then s / ε is also an upper bound for the set A. But this is impossible, since s is the smallest upper bound for A.

To prove the converse, assume that the number s satisfies the conditions (i) and (ii). By the condition (i), s is an upper bound for the set A. If s ≠ sup A, then s > sup A and $\varepsilon = s / \sup A > 1$. By the condition (ii), there exists at least a number a ∈ A such that $|s / a| < \varepsilon$. By the definition of the number ε, this means that a > sup A. This is impossible, hence s = sup A.

Theorem 1.16.[9] Let $\{x_n\}$ and $\{y_n\}$ be multiplicative Cauchy sequences in a multiplicative metric space (X, d). The sequence $\{d(x_n, y_n)\}$ is also a multiplicative Cauchy sequence in the multiplicative metric space $(\mathbb{R}_+, d^*)$.

Proof. From the multiplicative reverse triangle inequality, we have

$$d^*\left(d(x_n, y_n),\, d(x_m, y_m)\right) = \left|\frac{d(x_n, y_n)}{d(x_m, y_m)}\right|$$

$$\leq \left|\frac{d(x_n, y_n)}{d(x_m, y_n)}\right| \left|\frac{d(x_m, y_n)}{d(x_m, y_m)}\right|$$

$$\leq d(x_n, x_m) \cdot d(y_n, y_m).$$

Since $\{x_n\}$ and $\{y_n\}$ are multiplicative Cauchy sequences, for any $\varepsilon > 1$, there exists N ∈ N such that $d(x_n, x_m) < \sqrt{\varepsilon}$ and $d(y_n, y_m) < \sqrt{\varepsilon}$ for all $n, m \geq N$. This implies $d^*(d(x_n, y_n), d(x_m, y_m)) < \varepsilon$ for all $n, m \geq N$, which says $\{d(x_n, y_n)\}$ is a multiplicative Cauchy sequence.

3. MAIN RESULT

Theorem 2.1. If S and T are self-mappings defined on a complete multiplicative metric space (X, d) satisfying the condition,

$$(2.1) \quad d(Sx, Ty) \leq \left\{\max\left\{d(x, y), d(x, Sx), d(y, Sx), \frac{d(x, Sx)\, d(y, Ty)}{1 + d(Sx, Ty)}, d(x, Ty), d(y, Ty)\right\}\right\}^{\lambda}$$

$\forall x, y \in X$ and $\lambda \in [0,1/2]$, then S and T have a unique common fixed point.

Proof: Let x_0 be an arbitrary in X. Since $S(X) \subseteq X$ and $T(X) \subseteq X$, we construct the sequence $\{x_n\}$ in X, such that

(2.2) $\quad x_{2n+1} = Sx_{2n}$ and $x_{2n+2} = Tx_{2n+1} \quad \forall n \geq 0$

from the definition of $\{x_n\}$ and using (2.1), we have

$$d(x_{2n+1}, x_{2n+2}) = d(Sx_{2n}, Tx_{2n+1})$$

$$\leq \left\{ \max \left\{ \begin{array}{l} d(x_{2n}, x_{2n+1}), d(x_{2n}, Sx_{2n}), d(x_{2n+1}, Sx_{2n}), \\ \dfrac{d(x_{2n}, Sx_{2n}) d(x_{2n+1}, Tx_{2n+1})}{1 + d(Sx_{2n}, Tx_{2n+1})}, d(x_{2n}, Tx_{2n+1}), d(x_{2n+1}, Tx_{2n+1}) \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ \max \left\{ \begin{array}{l} d(x_{2n}, x_{2n+1}), d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+1}), \dfrac{d(x_{2n}, x_{2n+1}) d(x_{2n+1}, x_{2n+2})}{1 + d(x_{2n+1}, x_{2n+2})}, \\ d(x_{2n}, x_{2n+2}), d(x_{2n+1}, x_{2n+2}) \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ \max \left\{ d(x_{2n}, x_{2n+1}), d(x_{2n}, x_{2n+1}), 1, d(x_{2n}, x_{2n+1}) d(x_{2n}, x_{2n+2}), d(x_{2n+1}, x_{2n+2}) \right\} \right\}^{\lambda}$$

$$\text{since } \left\{ 1 + d(x_{2k}, x_{2k+1}) > d(x_{2k}, x_{2k+1}) \right\}$$

$$d(x_{2n+1}, x_{2n+2}) \leq d^{\lambda}(x_{2n}, x_{2n+2})$$

$$d(x_{2n+1}, x_{2n+2}) \leq d^{\lambda}(x_{2n}, x_{2n+1}) d^{\lambda}(x_{2n+1}, x_{2n+2})$$

$$(2.3) \quad d(x_{2n+1}, x_{2n+2}) \leq d^{\lambda/1-\lambda}(x_{2n}, x_{2n+1})$$

Now

$$d(x_{2n}, x_{2n+1}) = d(x_{2n+1}, x_{2n}) = d(Sx_{2n}, Tx_{2n-1})$$

$$\leq \left\{ \max \left\{ \begin{array}{l} d(x_{2n}, x_{2n-1}), d(x_{2n}, Sx_{2n}), d(x_{2n-1}, Sx_{2n}), \dfrac{d(x_{2n}, Sx_{2n}) d(x_{2n-1}, Tx_{2n-1})}{1 + d(Sx_{2n}, Tx_{2n-1})}, \\ d(x_{2n}, Tx_{2n-1}), d(x_{2n-1}, Tx_{2n-1}) \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ \max \left\{ \begin{array}{l} d(x_{2n}, x_{2n-1}), d(x_{2n}, x_{2n+1}), d(x_{2n-1}, x_{2n+1}), \dfrac{d(x_{2n}, x_{2n+1}) d(x_{2n-1}, x_{2n})}{1 + d(x_{2n+1}, x_{2n})}, \\ d(x_{2n}, x_{2n}), d(x_{2n-1}, x_{2n}) \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ \max \left\{ \begin{array}{l} d(x_{2n}, x_{2n-1}), d(x_{2n}, x_{2n+1}), d(x_{2n-1}, x_{2n+1}), d(x_{2n-1}, x_{2n}), \\ d(x_{2n}, x_{2n}), d(x_{2n-1}, x_{2n}) \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ \max \left\{ d(x_{2n}, x_{2n-1}), d(x_{2n}, x_{2n+1}), d(x_{2n-1}, x_{2n}) d(x_{2n}, x_{2n+1}), d(x_{2n-1}, x_{2n}), 1, d(x_{2n-1}, x_{2n}) \right\} \right\}^{\lambda}$$

$$d(x_{2n}, x_{2n+1}) \leq \left\{ d(x_{2n-1}, x_{2n}) d(x_{2n}, x_{2n+1}) \right\}^{\lambda}$$

$$d(x_{2n},x_{2n+1}) \le d^{\lambda}(x_{2n-1},x_{2n})\, d^{\lambda}(x_{2n},x_{2n+1})$$

(2.4) $d(x_{2n},x_{2n+1}) \le d^{\lambda/1-\lambda}(x_{2n-1},x_{2n})$

From (2.3) & (2.4), we have

(2.5) $d(x_n,x_{n+1}) \le d^{h}(x_{n-1},x_n)$ for all n $where\ h = \dfrac{\lambda}{1-\lambda}$

using (2.5), we have

$$d(x_n,x_{n+1}) \le d^{h}(x_{n-1},x_n) \le d^{h^2}(x_{n-2},x_{n-1}) \le \ldots\ldots\ldots\ldots \le d^{h^n}(x_0,x_1)$$

(2.6) $d(x_n,x_{n+1}) \le d^{h^n}(x_0,x_1)\ for\ all\ n$

for m > n and using (2.6) then we have,

$$\begin{aligned} d(x_m,x_n) &\le d(x_m,x_{m-1})\, d(x_{m-1},x_{m-2}) --- d(x_{n+1},x_n) \\ &\le d^{h^{m-1}}(x_1,x_0)\, d^{h^{m-2}}(x_1,x_0) --------- d^{h^n}(x_1,x_0) \\ &\le d^{h^{m-1}+h^{m-2}+-----------h^n}(x_1,x_0) \end{aligned}$$

$$d(x_m,x_n) \le d^{h^n/1-h}(x_1-x_0)$$

$\Rightarrow d(x_m,x_n) \to 1\ as\ (m,n\to\infty)$. Hence $\{x_n\}$ is a multiplicative Cauchy sequence.

By the completeness of X, $\exists\ z \in X\ such\ that\ x_n \to z\ as\ (n\to\infty)$

Moreover, we will show that $Sz = z$ By the notion of a complete multiplicative metric space.

$$\begin{aligned} d(z,Sz) &\le d(z,x_{2k+2})\, d(x_{2k+2},Sz) \\ &= d(z,x_{2k+2})\, d(Sz,x_{2k+2}) \\ &= d(z,x_{2k+2})\, d(Sz,Tx_{2k+1}) \qquad \text{[using (2.2)]} \\ &\le d(z,x_{2k+2})\left\{\max\left\{\begin{array}{l} d(z,x_{2k+1}), d(z,Sz), d(x_{2k+1},Sz), \dfrac{d(z,Sz)\,d(x_{2k+1},Tx_{2k+1})}{1+(Sz,Tx_{2k+1})}, \\ d(z,Tx_{2k+1}), d(x_{2k+1},Tx_{2k+1}) \end{array}\right\}\right\}^{\lambda} \\ &\le d(z,x_{2k+2})\left\{\max\left\{\begin{array}{l} d(z,x_{2k+1}), d(z,Sz), d(x_{2k+1},Sz), \dfrac{d(z,Sz)\,d(x_{2k+1},x_{2k+2})}{1+(Sz,x_{2k+2})}, \\ d(z,x_{2k+2}), d(x_{2k+1},x_{2k+2}) \end{array}\right\}\right\}^{\lambda} \end{aligned}$$

Taking $k\to\infty$, we have

$$d(z,Sz) \le d(z,z)\left\{\max\left\{d(z,z), d(z,Sz), d(z,Sz), \frac{d(z,Sz)\,d(z,z)}{1+d(Sz,z)}, d(z,z), d(z,z)\right\}\right\}^{\lambda}$$

$$\le 1\{\max\{1, d(z,Sz), d(z,Sz), 1, 1, 1\}\}^{\lambda}$$

$$d(z,Sz) \le d^{\lambda}(z,Sz) \text{ a contradiction since } \lambda \in [0,1/2]$$

which implies that $d(Sz,z)=1$

(2.7) Hence $Sz = z$

Now we will show that

$Tz = z$

$$d(z,Tz) \le d(z,x_{2k+1})\, d(x_{2k+1},Tz)$$

$$\le d(z,x_{2k+1})\, d(Sx_{2k},Tz) \qquad \text{[using (2.2)]}$$

$$\le d(z,x_{2k+1})\left\{\max\left\{\begin{array}{l} d(x_{2k},z), d(x_{2k},Sx_{2k}), d(z,Sx_{2k}), \dfrac{d(x_{2k},Sx_{2k})\,d(z,Tz)}{1+d(Sx_{2k},Tz)}, \\ d(x_{2k},Tz), d(z,Tz) \end{array}\right\}\right\}^{\lambda}$$

$$\le d(z,x_{2k+1})\left\{\max\left\{\begin{array}{l} d(x_{2k},z), d(x_{2k},x_{2k+1}), d(z,x_{2k+1}), \dfrac{d(x_{2k},x_{2k+1})\,d(z,Tz)}{1+d(x_{2k+1},Tz)}, \\ d(x_{2k},Tz), d(z,Tz) \end{array}\right\}\right\}^{\lambda}$$

Taking k $\to\infty$, we have

$$d(z,Tz) \le d(z,z)\left\{\max\left\{d(z,z), d(z,z), d(z,z), \frac{d(z,z)\,d(z,Tz)}{1+d(z,Tz)}, d(z,Tz), d(z,Tz)\right\}\right\}^{\lambda}$$

$$\le 1\left\{\max\left\{1,1,1,1, d(z,Tz), d(z,Tz)\right\}\right\}^{\lambda}$$

$$d(z,Tz) \le d^{\lambda}(z,Tz) \text{ a contradiction since } \lambda \in [0,1/2]$$

Hence, we have

$d(z,Tz)=1$ i.e $Tz = z$

(2.8) Tz = z

using (2.7) and (2.8), we have

(2.9) $Sz = Tz = z$

Hence z is a fixed point of S and T.

Uniqueness :

Let w $\neq$ z and w is fixed point of S and T

Now

$$d(z,w) = d(Sz,Tw)$$

$$\le \left\{\max\left\{d(z,w), d(z,Sz), d(w,Sz), \frac{d(z,Sz)\,d(w,Tw)}{1+d(Sz,Tw)}, d(z,Tw), d(w,Tw)\right\}\right\}^{\lambda}$$

$$\leq \left\{ \max\left\{ d(z,w), d(z,z), d(w,z), \frac{d(z,z)d(w,w)}{1+d(z,w)}, d(z,w), d(w,w) \right\} \right\}^{\lambda}$$

$$\leq \left\{ \max\left\{ d(z,w), 1, d(w,z), \frac{1}{1+d(z,w)}, d(z,w), 1 \right\} \right\}^{\lambda}$$

$d(z,w) \leq d^{\lambda}(z,w)$ *a contradiction* since $\lambda \in [0,1/2]$

$\Rightarrow d(z,w) = 1$

i.e $z = w$ Hence proved.

Corollary: If T is a self mapping defined on a complete multiplicative metric space (X,d) satisfying the condition

$$d(Tx,Ty) \leq \left\{ \max\left\{ d(x,y), d(x,Tx), d(y,Tx), \frac{d(x,Tx)d(y,Ty)}{1+d(Tx,Ty)}, d(x,Ty), d(y,Ty) \right\} \right\}^{\lambda}$$

and $\lambda \in [0,1/2]$ $\forall\ x, y \in X$ .Then T has a unique fixed point.

Proof: we can prove the result applying in last theorem setting S=T.

REFERENCES

[1] A.E. Bashirov, E.M. Kurpınar and A. Ozyapıcı, Multiplicative calculus and its applications, J. Math.Analy. App., 337(2008) 36-48.

[2] Chakkrid Klin-eam and Cholatis Suanoom, Some Common Fixed-Point Theorems for Generalized-Contractive-Type Mappings on Complex-Valued Metric Spaces, Abstr. Appl. Anal.

[3] Volume 2013 (2013), Article ID 604215, 6 pages.

[4] Muttalip Ŏzavsar and Adem C. Cevikel, Fixed points of multiplicative contraction mappings on multiplicative metric space, Mathematics Subject Classification, 1991

[5] R.P.Agarwal, M. A. El-Gebeily and D. O'Regan, Generalized contractions in partially ordered metric spaces, Applicable Analysis. 87(2008), 109-116

[6] S. A. Mohiuddine, M. Cancan and H. Sevli, Intuitionistic fuzzy stability of a Jensen functional equation via fixed point technique, Math. Comput. Model. 54 (2011), 2403-2409.

[7] S.B.Nadler, Multivalued nonlinear contraction mappings, Pacific J.Math. 30(1969) 475-488. [8] Soon-Mo Jung, A Fixed Point Approach to the Stability of Differential Equations y′ = F(x, y),Bull of the Malys. Math.Sci. Soc. (33) (2010), 47-56

[8] W.Takahashi, Nonlinear Functional Analysis:Fixed point theory and its applications, Yokohama Publishers, 2000

[9] Y. Kimura and W. Takahashi, Weak convergence to common fixed points of countable nonexpansive mappings and its applications, Journal of the Korean Mathematical Society 38 (2001), 1275-1284.

[10] Y. Kimura, Convergence of a Sequence of Sets in a Hadamard Space and the Shrinking Projection Method for a Real Hilbert Ball, Abstract and App. Analy., Article ID 582475 2010

[11] T.Suzuki, Subrahmanyam's fixed point theorem, Nonlinear Analysis, 71(2009)1678-1683.